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An overview of quantitative risk measures for loss of life and economic damage

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Abstract

A comprehensive overview of methods to quantify and limit risks arising from different sources is still missing in literature. Therefore, a study of risk literature was carried out by the authors. This article summarises about 25 quantitative risk measures. A risk measure is defined as a mathematical function of the probability of an event and the consequences of that event. The article focuses mainly on risk measures for loss of life (individual and societal risk) and economic risk, concentrating on risk measurement experiences in The Netherlands. Other types of consequences and some international practices are also considered. For every risk measure the most important characteristics are given: the mathematical formulation, the field of application and the standard set in this field. Some of the measures have been used in a case study to calculate the flood risks for an area in The Netherlands.

Keywords: Risk measures; Quantitative risk assessment; Societal risk; Flood risk; Economic risk

1. Introduction

Human existence involves exposure to many hazards. Natural disasters such as floods and earthquakes cost thousands of lives every year all over the world. Since the industrial revolution, technical hazards, such as aeroplane crashes, train derailments, tunnel fires and industrial accidents also disrupt society on a regular basis.

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Long ago, people tried to guard themselves from natural hazards with relatively simple methods, for example by building their houses on high grounds to protect them against floods. As society changed protection systems were built, such as dams and dikes. Later, new technological inventions, including nuclear power and aviation, and their accompanying hazards were introduced. Other developments, such as population growth and growing levels of production, consumption and transportation, have lead to an increase of hazards and of the consequences of accidents. Nowadays, large amounts of money are spent to protect society against these disasters. However, in decision and policy making these expenditures on safety have to compete with other public interests, for instance public health and the development of new infrastructure.

It is important to realise that decision-making regarding risks is very complex and that not only technical aspects but also political, psychological and social processes all play an important role. In this complex decision-making process a clear identification of the risks and of the effects of risk reduction measures is very useful. From a technical point of view, the extent of the risks and the effects of risk reduction measures can be quantified in a quantitative risk assessment (QRA). Thus, the QRA can provide a basis for rational decision-making regarding risks. Generally four phases are distinguished in literature on quantitative risk assessment, see for example Vrouwenvelder et al. [1].

- *Qualitative analysis*: Definition of the system and the scope, identification and description of the hazards, failure modes and scenarios.
- *Quantitative analysis*: Determination of the probabilities and consequences of the defined events. Quantification of the risk in a risk number or a graph as a function of probabilities and consequences.
- *Risk evaluation*: Evaluation of the risk on grounds of the results of the former analyses. In this phase the decision is made whether or not the risk is tolerable.
- *Risk control and risk reduction measures*: Depending on the outcome of the risk evaluation, measures may have to be taken to reduce the risk. It should also be determined how the risks can be controlled (for example by inspection, maintenance or warning systems).

Risk measures play an important role in communicating the whole risk assessment process. A risk measure is defined as a mathematical function of the probability of an event and the consequences of that event. This risk measure constitutes the basis for evaluation of risks by the decision-makers. Limits or standards set an acceptable risk level. Finally, the risk measure can be used as an instrument to show the effect of risk reducing actions.

1.1. Objective of this study

In the study of flood risk in The Netherlands it has been found that a comprehensive overview of methods to quantify and limit risks arising from different sources is still missing in literature. Therefore, the authors carried out a study of risk literature aimed at giving an overview of quantitative risk measures. Measures that deal with the risk qualitatively were not considered. This study concentrates on Dutch risk measurement experiences, mainly in the areas external safety and flood risk management. However, in order to give a more complete overview, some measures used in other countries are also included. This article focuses mainly on risk measures that consider loss of life and economic damage as a consequence. Some other types of risk are also described, for instance dealing with environmental risks. The risk measures are categorised according to the consequences they consider:

- Fatalities:
  - Individual risk (Section 2).
  - Societal risk (Section 3).
- Economic damage (Section 4).
- Environmental damage (Section 5).
- Integrated risk measures: considering various types of consequences (Section 7).
- Potential damage (Section 6).

The most important characteristics of the risk measures are described from a technical perspective for every category. Every section starts with an overview of the risk measures and their mathematical expressions. Consequently, the fields of application and the standards used are described. A specific problem in risk assessment is the monetary valuation of human life. A summary of available methods is given in Section 8. To show the possible applications, Section 9 discusses a case study, in which the flood risks have been calculated for an existing area in The Netherlands, using some of the risk measures described. The article is concluded in Section 10, which summarises the risk measures and their most important characteristics in a table. Finally, an evaluation of some important aspects is included.

2. Individual risk

2.1. Individual risk measures

The first measure is the individual risk (IR), as used by the Dutch Ministry of Housing, Spatial Planning and Environment (VROM). It is defined as the probability that an average unprotected person, permanently present at a certain location, is killed due to an accident resulting from a hazardous activity [2].

$$\mathrm{IR} = P_\mathrm{f} P_{\mathrm{d|f}}$$

where P_f is the probability of failure and $P_{\mathrm{d|f}}$ probability of dying of an individual in the case of failure, assuming the permanent unprotected presence of the individual.

The IR is thus a property of the place and as such it is useful in spatial planning. A slightly different definition, which considers whether or not the individual is actually present, is used by the Dutch Technical Advisory Committee on Water Defences (TAW) [3] and by Bohnenblust [4].

Bedford and Cooke [5] give an overview of measurements to express the individual risk. Besides the individual risk, as mentioned above, four other expressions are described. The *loss of life expectancy* shows the decrease of life expectancy due to various causes. The *delta yearly probability of death* computes the intensity at which a given activity is to be performed (in suitable units) to increase the yearly probability of death by 10^{-6}. The *activity specific hourly mortality rate* reflects the probability per time unit while engaged in a specified activity. An example is the fatal accident failure rate (FAFR) which gives the number of fatalities per 1000 h of exposure to a certain risk. A variant is the *death per unit activity*, which replaces the time unit by a unit measuring the amount of activity. The risks of travel by car, train or aeroplane are often expressed as the number of deaths per kilometre travelled.

A different definition is used by the UK's health and safety executive (HSE). According to this body, the individual risk is the risk that a typical user of a development is exposed to a dangerous dose or worse of toxic substance, heat or blast overpressure [6]. A dangerous dose is likely to cause the person severe distress or injury, but it does not lead to certain death. Given the different definition, this article adopts an alternative notation for the individual risk as defined by HSE: $\mathrm{IR}_{\mathrm{HSE}}$.

2.2. Fields of application and standards

In The Netherlands, the measure of individual risk is used to determine the risks of hazardous installations, transport routes and airports. Locations with equal individual risk levels are shown on a map with so-called risk contours that facilitate land use planning applications. Fig. 1 shows the typical risk contours for a hazardous installation and a transport route.

To limit the risks, the Dutch Ministry of Housing, Spatial planning and Environment (VROM) has set the following standard for populated areas [2].

$$\mathrm{IR} < 10^{-6}(\text{per year})$$

Risks lower than 10^{-6} per year should always be reduced to a level which is as low as reasonably achievable (ALARA). This standard is set for more or less involuntary imposed risks related to the locating of hazardous activities. The method of TAW [3] gives the opportunity to limit a broader set of risks, ranging from voluntary activities, such as mountaineering, to more involuntary risks, such as those of hazardous installations. The

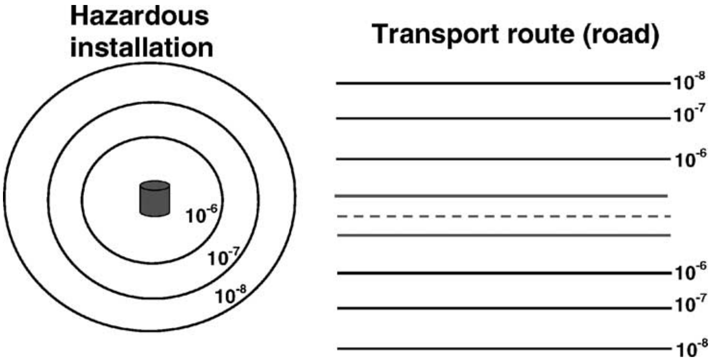


Fig. 1. Characteristic individual risk contours for a hazardous installation (point source) and a transport route (line source).

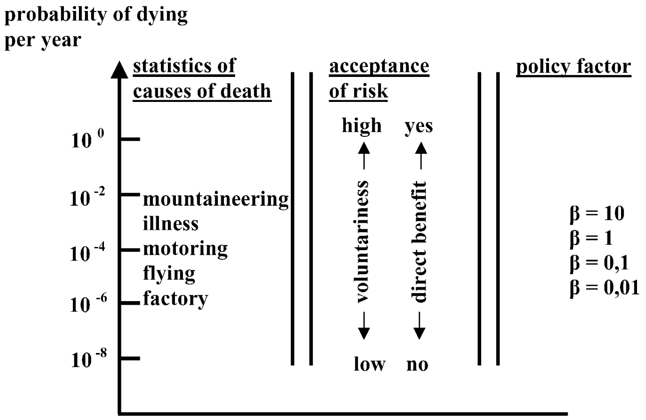


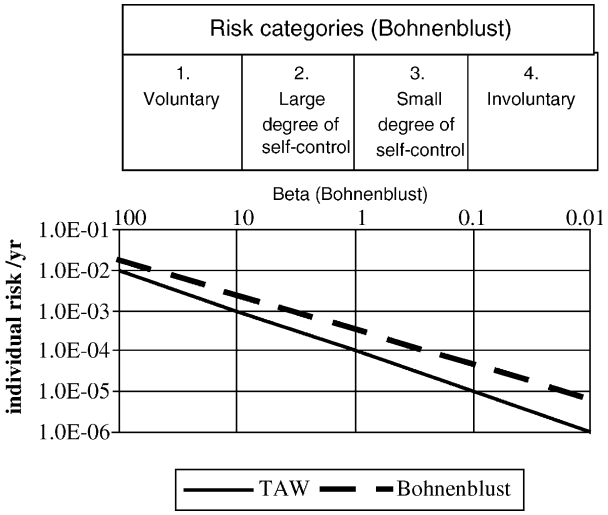
Fig. 2. Policy factor for different activities and various degrees of voluntary participation and benefit [7].

following standard is proposed by TAW:

$$\mathrm{IR} < \beta \cdot 10^{-4}(\text{per year})$$

In this expression the value of the policy factor β varies according to the degree to which participation in the activity is voluntary and with the perceived benefit. In Fig. 2 some β values are proposed for different activities. This method has been used in case studies concerning various risks in [7].

The background of the standard proposed by Bohnenblust [4] is comparable with the TAW standard. Bohnenblust limits the acceptable IR, taking into account the extent to which participation in an activity is voluntary and the degree of self-control in the activity.



Risk categories (Bohnenblust)			
1. Voluntary	2. Large degree of self-control	3. Small degree of self-control	4. Involuntary

Fig. 3. Individual risk standard according to Bohnenblust and TAW.

Four risk categories have been determined, ranging from voluntary to involuntary. The proposed limits of Bohnenblust and TAW are shown in Fig. 3. Bohnenblust studies the safety of the railway system in Germany [4].

HSE uses a framework for judging the tolerability of risks, considering an unacceptable, a tolerable and a broadly acceptable region [8]. Using HSE's definition of individual risk (IR_{HSE}), given in the last section, an IR_{HSE} of 10^{-6} should be used as a guideline for the boundary between the broadly acceptable and the tolerable regions for both workers and the public. For the boundary between the tolerable and the unacceptable regions no widely applicable criterion is given. However, an HSE document on the tolerability of risks in nuclear stations [9] suggests IR_{HSE} values of 10^{-3} for workers and 10^{-5} for the members of the public, as a boundary between the tolerable and the acceptable regions.

3. Societal risk

Ichem [10] defined societal risk as "the relationship between frequency and the number of people suffering from a specified level of harm in a given population from the realisation of specified hazards". Where individual risk gives the probability of dying on a certain location, the societal risk gives a number for a whole area, no matter precisely where the harm occurs within that area. The difference is shown in Fig. 4.

3.1. Societal risk measures

The aggregated weighted risk (AWR) as described by Piers [12] is calculated by multiplying the number of houses inside a certain area with their IR level:

$$\mathrm{AWR} = \iint_A \mathrm{IR}(x, y)\, h(x, y)\, \mathrm{d}x\mathrm{d}y$$

where IR(x,y) is the individual risk on location (x,y); h(x,y) number of houses on location (x,y) and A is area for which the AWR is determined.

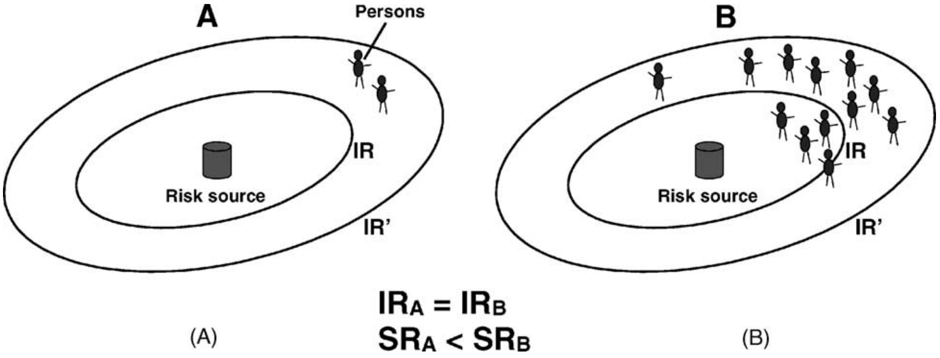


Fig. 4. The difference between individual and societal risk. Both situations have equal individual risk levels (shown by IR′ and IR). Because of the larger population density of situation B, B has a larger societal risk (based on [11]).

By integrating the individual risk levels and the population density the expected value of the number of fatalities can be determined [13]:

$$E(N) = \iint_A \mathrm{IR}(x, y)\, m(x, y)\, \mathrm{d}x\mathrm{d}y$$

where $E(N)$ is the expected value of the number of fatalities per year and $m(x,y)$ is the population density on location (x,y).

The scaled risk integral (SRI), as defined by Carter [14], takes the individual risk level and other characteristics of the location into account:

$$\mathrm{SRI} = \frac{P\, \mathrm{IR}_{\mathrm{HSE}} T}{A}, \quad \text{where } P = \frac{n + n^2}{2}$$

where $\mathrm{IR}_{\mathrm{HSE}}$ is the individual risk per million year (cpm), as defined by HSE (see Section 2.1); T the share of time the area is occupied by n persons; A the surface of the area in hectares; P the population factor and n is the number of persons in the area. Note that the SRI is not dimensionless: $(\mathrm{person} + \mathrm{person}^2)/(10^6\ \mathrm{ha\ year})$.

The first three expressions are based on the individual risk contours. Other societal risk measures can be derived from the probability density function (pdf) of the number of fatalities per year. Although, individual and societal risk calculations are often based on the same data, no mathematical relation has yet been found between the individual risk contours and the pdf of the number of fatalities. Simultaneous individual and societal risk calculations are therefore often carried out with numerical methods.

Societal risk is often represented graphically in a FN-curve. This curve displays the probability of exceedance as a function of the number of fatalities, on a double logarithmic scale.

$$1 - F_N(x) = P(N > x) = \int_x^{\infty} f_N(x)$$

where $f_N(x)$ is the probability density function (pdf) of the number of fatalities per year; $F_N(x)$ the probability distribution function of the number of fatalities per year, signifying the probability of less than x fatalities per year.[1]

A simple measure for societal risk is the expected value of the number of fatalities per year, $E(N)$, in literature often referred to as the potential loss of life (PLL):

$$E(N) = \int_0^{\infty} x\, f_N(x)\, \mathrm{d}x$$

Ale et al. [15] proposes the area under the FN-curve as a measure for societal risk. Vrijling and van Gelder [16] have shown that this measure equals the expected number of fatalities per year:

$$\int_0^{\infty} (1 - F_N(x))\, \mathrm{d}x = \int_0^{\infty} \int_x^{\infty} f_N(u)\, \mathrm{d}u\, \mathrm{d}x = \int_0^{\infty} \int_x^{u} f_N(u)\, \mathrm{d}x\, \mathrm{d}u$$
$$= \int_0^{\infty} u f_N(u)\, \mathrm{d}u = E(N)$$

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[1] Note that a different convention can be found in other published works, in which the symbol $F_N(x)$ (or $F(x)$) signifies the probability of "x or more" fatalities per year.

The British Health and Safety Executive (HSE) has defined a risk integral as a measure for societal risk [17]:

$$\mathrm{RI} = \int_0^\infty x(1 - F_N(x))\,\mathrm{d}x$$

Vrijling and van Gelder [16] mathematically proved that the RI can be expressed in two characteristics of the pdf of the number of fatalities, the expected value $E(N)$ and the standard deviation $\sigma(N)$:

$$\mathrm{RI} = \tfrac{1}{2}(E^2(N) + \sigma^2(N))$$

HSE [18] defined a weighted risk integral parameter called the Risk integral (COMAH) ($\mathrm{RI}_{\mathrm{COMAH}}$):

$$\mathrm{RI}_{\mathrm{COMAH}} = \int_0^\infty x^\alpha f_N(x)\,\mathrm{d}x$$

The aversion to accidents with many fatalities is represented by a coefficient α, which is ≥ 1. Based on an analysis of situations that are expected to occur in practice a value of $\alpha = 1.4$ was chosen for the risk aversion coefficient. Smets [11] proposed a similar measure:

$$\int_1^{1000} x^\alpha f_N(x)\,\mathrm{d}x$$

If the integration boundaries are not taken into account, the expression of Smets and the $\mathrm{RI}_{\mathrm{COMAH}}$ both equal the expected value for $\alpha = 1$. If $\alpha = 2$ the expressions equal the second moment of the pdf:

$$\int x^2 f_N(x)\,\mathrm{d}x = E(N^2)$$

$$E(N^2) = E^2(N) + \sigma^2(N)$$

Bohnenblust [4] presents the perceived collective risk R_p as a measure for societal risk:

$$R_\mathrm{p} = \int_0^\infty x\,\varphi(x)\,f_N(x)\,\mathrm{d}x$$

where $\phi(x)$ is the risk aversion, a function of the number of fatalities x.

For this measure the expected value of the number of fatalities is weighed with a risk aversion function $\phi(x)$. From the risk aversion values proposed by Bohnenblust it can be deduced that $\varphi(x) \approx \sqrt{0.1x}$ [16]. The expression can now be written as follows:

$$R_\mathrm{p} = \int_0^\infty \sqrt{0.1}\,x^{1.5}\,f_\mathrm{N}(x)\,\mathrm{d}x$$

Kroon and Hoej [19] propose a similar measure, the expected disutility of a system U_{sys}:

$$U_{\mathrm{sys}} = \int_0^\infty x^\alpha P(x)\,f_N(x)\,\mathrm{d}x$$

Again the weighting factor α has been included together with a risk aversion factor $P(x)$, which shows the expected disutility as a function of the number of fatalities. Note that the risk integral, the $\mathrm{RI}_{\mathrm{COMAH}}$ and the measures proposed by Smets, Bohnenblust and Kroon and Hoej are all expected (dis)utility measures, which can all be described with the following general formula.

$$\int x^{\alpha} C(x)\, f_N(x)\, \mathrm{d}x$$

Different authors have chosen different values of α (ranging from 1 to 2) and of the factor C, which is a constant or a function of x.

The measure of total risk, as proposed by Vrijling et al. [7], is composed of the expected value of the number of fatalities and the standard deviation, which is multiplied by a risk aversion factor k:

$$\mathrm{TR} = E(N) + k\sigma(N)$$

The total risk takes a risk aversion index k and the standard deviation into account and is therefore called risk averse. The standard deviation is relatively high for accidents with low probabilities and high consequences.

A quick overview of the societal risk measures reveals that two types of expressions can be distinguished. The FN-curve and the expected value are directly derived from the pdf and are therefore called risk neutral. Risk aversion can be modelled by weighing the expected value with a factor α, taking into account a risk aversion factor ($P(x)$ or $\varphi(x)$) or by involving the standard deviation in the equation ($\alpha = 2$).

3.2. *Fields of application and standards*

In the decision-making process regarding the risks of The Netherlands' national airport Schiphol the area under the FN-curve (=the expected value) was first proposed as a risk measure. After that, the AWR was put forward as measure for the risks. In the past an agreement was made that AWR levels were no longer allowed to increase, the so-called standstill principle [12].

No current use of the determination of the expected value according to the individual risk contours has been found, neither has a standard been proposed.

The SRI has been in use in the UK for many years as an advisory instrument for local planning authorities, for locating new hazardous installations. For the purposes of decision-making comparison values are used, for examples see [14].

The FN-curve, originally introduced for the assessment of the risks in the nuclear industry [20], is used in various countries to express and limit risks, predominantly of hazardous installations. As part of the Dutch external safety policy, the so-called group-risks are determined on a national level for various activities. These are shown in the FN-curve in Fig. 5.

In several countries a FN criterion line limits the risks of various hazardous activities. These standards can be described with the following general formula:

$$1 - F_N(x) < \frac{C}{x^n}$$

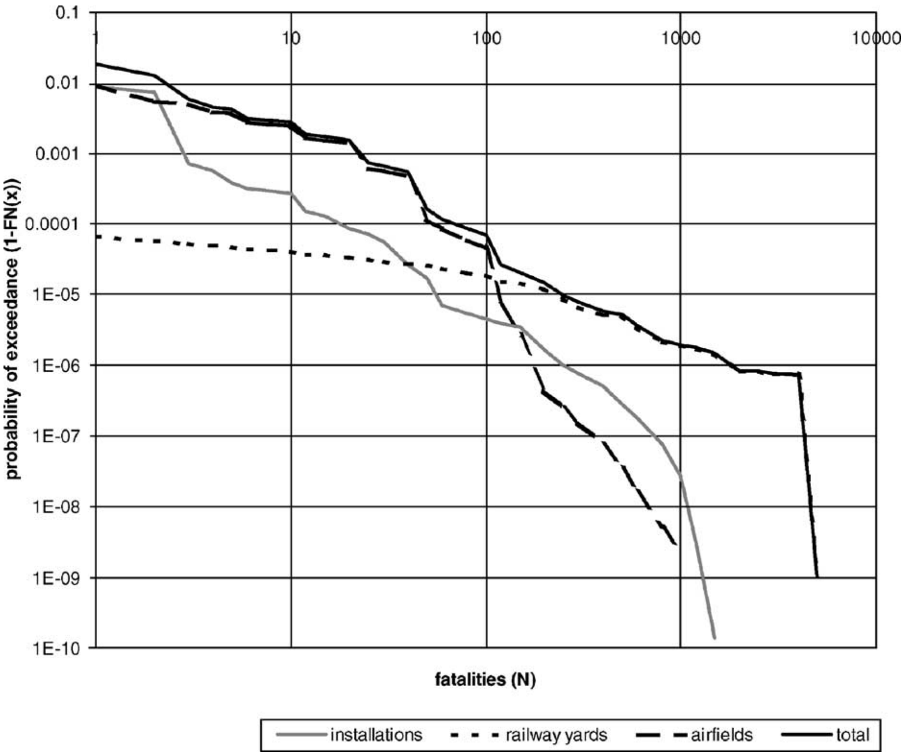

Fig. 5. FN-curve for the risks of various activities in The Netherlands in 1999 (source: RIVM, NL).

where n is the steepness of the limit line and C the constant that determines the position of the limit line.

A standard with a steepness of $n = 1$ is called risk neutral. If the steepness $n = 2$, the standard is called risk averse [16]. In this case larger accidents are weighted more heavily and are thus only accepted with a relatively lower probability. Table 1 gives the values of the coefficients for some international standards and the FN limit lines are shown in Fig. 6.

Commonly, as a part of the standard, an ALARA (or ALARP) region has been determined below the limit line, in which the risk should be reduced to a level that is as low as reasonably achievable (or possible).

Table 1
Some international standards limiting the FN-curve [2,8,11,21]

Country	n	C	Application
UK (HSE)	1	10^{-2}	Hazardous installations
Hong Kong (truncated)	1	10^{-3}	Hazardous installations
The Netherlands (VROM)	2	10^{-3}	Hazardous installations
Denmark	2	10^{-2}	Hazardous installations

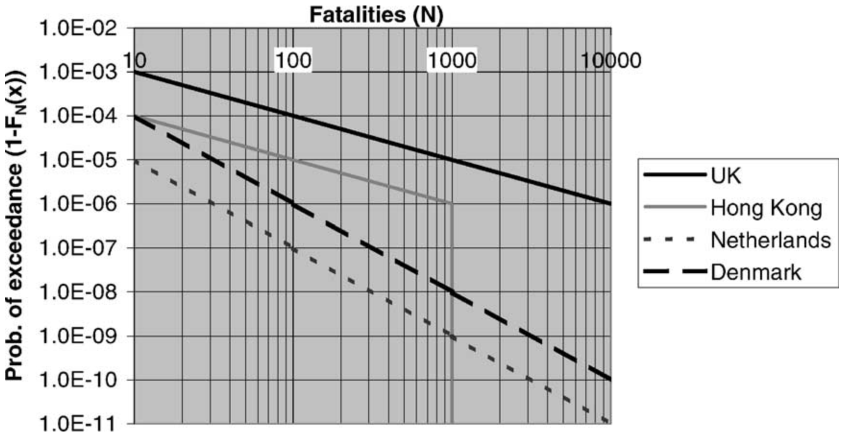

Fig. 6. Some international standards in FN format [2,8,11,21].

The expected number of fatalities is used in the regulation of the risks of dams. Standards have been proposed by British Columbia Hydro [22] and the United States Bureau of Reclamation [23].

$$\text{BC Hydro}: E(N) < 10^{-3} (\text{fatalities/year}),$$
$$\text{USBR}: E(N) < 10^{-2} (\text{fatalities/year})$$

A risk integral is used in the United Kingdom as a part of HSE's examination of proposals for determining the site for new hazardous installations. However, it has not yet been linked to a limiting value.

The RI_{COMAH} was developed to give an indication of the magnitude of societal risks and of risks in the vicinity of a major accident hazard installation. It is intended for use by the HSE as a first screening tool for the examination of safety reports submitted under regulations. Examples of the use of the methodology are given in [18]. Acceptability criteria for the value of the RI_{COMAH} can be derived by determining the RI_{COMAH} values that correspond to FN criterion lines.

In literature no current application of the method of Smets can be found. The proposed limit depends on the steepness of the utility function. For a quadratic utility function ($\alpha = 2$) the following limit has been proposed [11]:

$$\int_1^{1000} x^2 f_N(x) < 10^{-2}$$

No standard has been proposed for Bohnenblust's measure. It has been used as part of a method, which takes individual risk, societal risk and economic aspects into account in studies concerning the investments in railway safety in Germany [4].

Kroon and Hoej's measure has been used for evaluating the risk attitude for tunnels in an OECD/PIARC study concerning the transport of dangerous goods through tunnels [19].

Vrijling et al. [7] presented a standard for the acceptable risk. It limits the Total Risk on a national level, considering the policy factor β, which is has already been presented in

Section 2.2.

$$\mathrm{TR} < \beta \cdot 100, \qquad \mathrm{TR} = E(N) + k\sigma(N)$$

This national criterion for acceptable risk can be translated into a standard for a single installation or location. This criterion has the typical form of a FN limit, with a quadratic steepness ($\alpha = 2$):

$$1 - F_N(x) < \frac{C}{x^2}$$

Suppose that the expected value of the number of fatalities is much smaller than its standard deviation (which in general is true for accidents with low probabilities and high consequences) and assume a Bernoulli distribution. The factor C can now be written as a function of the number of installations on a national level (N_A), the risk aversion factor (k), and the policy factor (β) [7]:

$$C = \left[\frac{\beta \cdot 100}{k\sqrt{N_\mathrm{A}}}\right]^2$$

This equation leads to the conclusion that the Dutch limit for societal risk ($C = 10^{-3}$, $\alpha = 2$), as presented in Fig. 6, is a special case of the total risk limit described above. The method of total risk has been used in several case studies, for example in [7]. This paper shows that setting limits with FN-curves on a local level or on an installation level can lead to an undesired situation on a national level. If a risk criterion is defined on an installation level, the national criterion is determined by the number of locations. An increase in the number of installations, each of them acceptable according to the local limit, can therefore lead to an unacceptable high-risk level on a national scale. To prevent these problems it is proposed to set a limit on a national level and to distribute the acceptable risk over the locations.

A similar problem with the use of FN limit lines is illustrated by Evans and Verlander [24]. While the risks of installations are each acceptable by the FN limit line, the risks of the "probabilistic mixture" of these installations can exceed the limit, although the number of installations has not changed. Furthermore, Evans and Verlander conclude that the use of FN criterion lines can lead to unreasonable and inconsistent decisions and that the use of "expected (dis)utility functions" is therefore preferable.

4. Economic risk

Besides the danger of loss of life due to certain activities, the economic risks play an important role in decision-making. This section describes approaches to quantify economic risks and their applications.

4.1. *Economic risk measures*

A FD-curve displays the probability of exceedance as a function of the economic damage. The FD-curve and the expected value of the economic damage can be derived from the pdf of the economic damage ($f_D(x)$).

$$1 - F_D(x) = P(D > x) = \int_x^\infty f_D(x)\,\mathrm{d}x$$

$$E(D) = \int_0^\infty x\, f_D(x)\,\mathrm{d}x$$

where $F_D(x)$ is the probability distribution function of the economic damage and $E(D)$ is the expected value of the economic damage.

Analogous to the FN-curve and the expected number of fatalities (see Section 3.1), it can be shown that the area below the FD-curve equals the expected value $E(D)$.

The problem of the acceptable level of risk can also be formulated as an economic decision problem [25]. According to the method of economic optimisation, the total costs in a system (C_{tot}) are determined by the sum of the expenditure for a safer system (I) and the expected value of the economic damage. In the optimal economic situation the total costs in the system are minimised:

$$\min(C_{\mathrm{tot}}) = \min(I + E(D))$$

With this criterion the optimal probability of failure of a system can be determined, provided investments (I) and the expected economic damage ($E(D)$) are a function of the probability of failure. Slijkhuis et al. [26] showed how uncertainty and risk aversion can be modelled in the method of economic optimisation. Investments and economic damage are modelled as random parameters. The determination of the optimal level of protection takes the standard deviation of the total costs and a risk aversion factor (k) into account. The attitude towards uncertainty and the risk aversion can be varied by adjusting the value of the risk aversion factor k. The economic optimum is now found by:

$$\min(\mu(C_{\mathrm{tot}}) + k\sigma(C_{\mathrm{tot}}))$$

4.2. *Fields of application and standards*

Novgorodsky's study on the economic risks in the Russian region [27] is an example of the use of a FD-curve. Jansen [28] has tried to obtain a financial economic risk limit in the form of a FD-curve. However, research of the economic risks in various fields (nuclear energy, aviation, floods) did not lead to a consistent economic risk limit.

The expected value of the economic damage is used as part of cost benefit analyses of flood prevention measures in the UK [29] and in The Netherlands [30]. In both approaches the benefits of a measure are determined by calculating the expected value of the economic damage before and after implementing the measure. The difference between these two values is the benefit, which can be weighed against the costs of the measures. A limit for the expected economic damage per year for dams has been proposed by BC Hydro [31], in which the financial risks for one dam should not exceed:

$$E(D) < \text{US\$ } 10{,}000 \text{ (per year)}$$

The method of economic optimisation was originally applied by van Danzig [25] to determine the optimal level of flood protection (i.e. dike height) for Central Holland (this polder forms the economic centre of The Netherlands). The total investments in raising dikes (I_{tot}) are determined by the initial costs (I_0) and the variable costs (I'). The dike is raised X m, the difference between the new dike height (h) and the current dike height (h_0).

$$I_{\mathrm{tot}} = I_0 + I'X \quad \text{and} \quad X = h - h_0$$

The expected value of the economic damage can be calculated from the probability of flooding (P_{b}), the damage caused by the flood (D), and the discount rate (r'). The flood level h is assumed exponentially distributed with parameters A and B.

$$E(D) = \frac{P_{\mathrm{b}} D}{r'} \quad \text{and} \quad P_{\mathrm{b}} = \mathrm{e}^{-(h-A)/B}$$

Now the total costs are formulated as the sum of investments and the expected value of the economic damage. The economic optimum is found by minimising the total costs. The derivative of the total costs and the dike height leads to the optimal flooding probability ($P_{\mathrm{b,opt}}$) and the optimal dike height (h_{opt}).

$$P_{\mathrm{b,opt}} = \frac{I'Br'}{D} \quad \text{and} \quad h_{\mathrm{opt}} = A - B\ln(P_{\mathrm{b,opt}})$$

The method of economic optimisation has also been applied for the design of various hydraulic structures, for example for breakwaters in [32].

The economic optimisation, which takes uncertainty into account, has been in used in a case study to determine optimal dike height for Central Holland [26]. The investments and the economic damage caused by inundation are modelled as random parameters. These uncertainties cause considerable increases of the economic optimal failure probability and of the dike height (compared with the results of van Danzig).

5. Environmental risk

5.1. Environmental risk measures

NORSOK (the competitive standing of the Norwegian offshore sector) has proposed the probability of exceedance of the time needed by the ecosystem to recover from the damage as a measure for environmental risk [33]:

$$1 - F_T(x) = P(T > x) = \int_x^{\infty} f_T(x)\,\mathrm{d}x$$

where $F_T(x)$ is probability distribution function of the recovery time; $f_T(x)$ probability density function of the recovery time of the ecosystem.

The energetic impact index [34] is a measure for the amount of energy lost per year, expressed in Joules. This method regards man as a part of the ecosystem. The energy loss caused by injured and dead humans and animals can be expressed in Joules, just like any other damage to nature. According to this method human life is equivalent to a certain amount of energy, about 800 billion Joules. This results in the following formula:

$$\mathrm{GPP_{lost}} = \mathrm{EPP} + \mathrm{GPP}'\,T$$

where $\mathrm{GPP}_{\mathrm{lost}}$ is the effect on the ecosystem and humans in Joules; EPP the energy loss of the system; GPP' the amount of energy needed during period T for recovery of harmed organisms.

5.2. Fields of application and standards

Limits for the environmental risks of offshore activities are set by NORSOK. NORSOK demands that "The duration of environmental damage shall be insignificant in relation to the expected time between such damages". The following limit to determine the acceptable risk for oil platforms [33] is derived from the quoted criterion:

$$1 - F_T(x) < \frac{0.05}{x}$$

No use of the Energetic Impact Index has been found in literature.

6. Potential consequences

A specific category is made up of the measures that consider the (potential) consequences of a hazardous activity. The difference with the risk measures described in the foregoing sections is that these risk measures do not take into account the probability of an accident.

6.1. Risk measures

The number of people at risk (PAR) gives an impression of the magnitude of a disaster and shows the number of persons present in the disaster area.

$$\mathrm{PAR} = \iint_A m(x, y)\,\mathrm{d}x\,\mathrm{d}y$$

The F-PAR is similar to the FN-curve. It displays the probability of exceedance as a function of the people at risk (instead of the number of fatalities for the FN-curve). Arguably, the FPAR curve gives a better impression of societal disruption than the FN-curve. An accident with a relatively low number of fatalities can cause a lot of injuries or a large disruption of society. An example is the explosion of a firework factory in The Netherlands in May 2000. This disaster had an enormous societal impact. However, the number of 20 fatalities would be relatively insignificant in a FN-curve. Similar to the PAR, the potential economic damage can be calculated for the (potential) disaster area.

6.2. Field of application

The PAR is used as measure for the potential disaster in risk analyses in various fields, for example for dams. An example of the use of an FPAR curve for the assessment of dam safety is given by Khan and Jamal [35]. An example of the use of the potential economic damage is the maximal economic damage map that was made for flood prone areas in The Netherlands.

7. Integrated methods

In literature some methods were found to consider various types of consequences in one expression or framework. These “integrated” methods are described in the following section.

7.1. Integrated risk measures and methods

The framework presented by Bohnenblust [4] considers the individual risk, societal risk and economic aspects. The societal risk is expressed by the measure of perceived collective risk, as described in Section 3.1. The risk can be expressed in monetary terms by determining the willingness to pay for every scenario of consequences. This method can consider different types of consequences (fatalities, economic damage, environmental damage). The measure that shows the risk sum for different types of consequences is the monetary collective risk (R_m):

$$R_\mathrm{m} = \sum_{i-1}^{n} P_i C_i \, \varphi(C_i) \, \omega(i)$$

where P_i is the probability of scenario i; C_i the consequences of scenario i; $\varphi(C_i)$ the risk aversion as a function of the consequences C_i; $\omega(i)$ the willingness to pay for measures to prevent scenario i.

The Dutch Technical Advisory Committee on Water Defences (TAW) developed a framework for acceptable risk, which limits the individual, societal and economic risk [3]. In this framework three measures are presented, which have previously been described in this paper. Firstly, the individual risk is considered. Secondly, societal risk is expressed by the measure of total risk. Subsequently, the method of economic optimisation is used.

Merz et al. [36] presented a method which limits the risks for man, economy and environment. Nine classes of possible consequences are determined: fatalities, injured, evacuated, psychological damage, death of animals, polluted water, polluted groundwater, polluted soil, material damage. The extent of the damage in each category is linked to an index value, ranging from 0 to 1. A limit line has been established to determine the acceptable probability of an accident, this line is described further in Section 7.2.

7.2. Fields of application and standards

To study the safety of the transportation of dangerous goods, Deutsche Bahn AG (Germany) applied Bohnenblust’s framework [4]. The effectiveness of risk reduction measures can be judged by comparing the investments with the monetary collective risk (R_m). The costs and R_m values of different measures can be plotted in a graph. The optimal risk reduction curve is formed by the measures resulting in the largest risk reduction with the smallest investments (see Fig. 7).

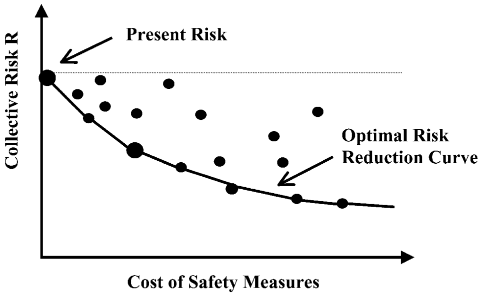

Fig. 7. Optimal risk reduction curve [4].

With the economical “marginal-cost criterion”, Bohnenblust shows that optimal safety is obtained in the point where the steepness of the curve is -1. The individual risk is evaluated by the limit presented in Section 2.2, Fig. 3.

The TAW framework has been applied in some case studies for flood prone areas in The Netherlands and the UK [3]. The individual risk associated with this method is limited to: $\text{IR} < \beta \cdot 10^{-4}$ (see Section 2.1). The total risk is limited with the corresponding limit: $\text{TR} < \beta \cdot 100$ (Section 3.2). An economic optimum is also determined with the method of economic optimisation. The most stringent of three criteria should be applied as the limit.

Merz’s method has been applied in the assessment of risks of the Swiss kanton Solothurn [36]. Fig. 8 shows the acceptable probability for an accident as a function of the index value. According to this method, the problem of choosing an acceptable risk level is transformed to the subjective determination of index values for different consequences.

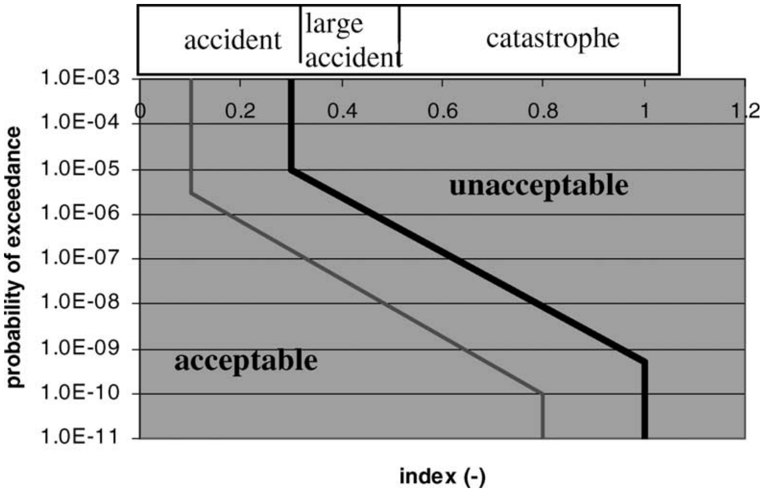

Fig. 8. Acceptable probability of an accident as a function of the index value. In the area between the two limit lines risk-reduction measures are necessary.

8. Economic valuation of human life

A specific problem in risk management is the monetary valuation of human life. Some people consider it unethical to put a price on human life. However, arguably, not taking the economic value of human life into account leads to a lower (economic) damage and thus results in a lower safety of the considered system. Some different methods of valuation of human life are found in literature on risk management and safety. This article distinguishes four approaches.

8.1. Macro economic valuation

The value of human life can be derived from (macro) economic units. According to the human capital approach, life is valued in proportion to a person’s potential economic production. Van Manen and Vrijling [37] proposed a method of valuation according to the Nett National Product. This approach was used in the economic optimisation in [38]. The total damage consists of the economic damage (D) and the economic value of (N) fatalities. Assume that every person has an economic value d. The economic optimum can again be found by minimising the total costs:

$$\min(C_{\text{tot}}) = \min(I + E(D + Nd))$$

This method was applied in the economic optimisation of polders in The Netherlands [38], resulting in an optimal flooding probability of:

$$P_{\text{b,opt}} = \frac{I'Br'}{D + Nd}$$

8.2. Comparative approach

In various fields of science expenditures concerning the protection of human life can be found. Some approaches relate the value of human life to the investment made and to the number of prevented fatalities. These approaches are therefore called comparative approaches. The cost of saving an extra life (CSX) expresses the investment made for saving one extra life. Vrijling and van Gelder [38] showed how the CSX can be determined for the method of economic optimisation. In the United States, a similar measure, the Absolute risk reduction index (ARRI), is determined in the risk assessment of dams [22].

The costs of saving an extra life year (CSXY) can be calculated by involving life expectancy in this method. An extensive study of CSXY values in various sectors has been performed by Tengs et al. [39]. This study showed that CSXY values vary widely across different sectors. As regards the use of CSX/CSXY values, it should be noted that the benefits of measures not only consist of the saving of human lives, but also of the prevention of damage in other fields (economics, environment). A better approach would be to look at societal decisions where the only benefit is an increase in human safety, thus a decrease of the probability of loss of life. Decisions in the field of public health can be of such nature. The effectiveness of medical treatments or precautions is often represented with quality adjusted life years (QALYs). One QALY is the increase of the life expectancy with a year of optimal quality. The investments in medicine can be related to gained QALYs, resulting in the costs per QALY.

A simple normative theory for the management of risks to the public is proposed by Lind [40]. A lifesaving alternative should return more years of life expectancy than years of work required to pay for the alternative. A lifesaving project, policy or regulation leads to an increase $\mathrm{d}h$ of life expectancy measured in QALYs. The measure requires an increment in wealth production, which requires an increment in work $\mathrm{d}w$. The efficiency of the project is the ratio $\mathrm{d}h/\mathrm{d}w$. If this efficiency is greater than 1 the project is acceptable to society. This method allows an assessment of various life saving alternatives on a common basis.

8.3. Utility based approach

The concept of utility is used in economics to model the behaviour of consumers and producers. It shows to what extent the needs of consumers have been satisfied. An example of the use of utility in risk analysis is the life quality method as developed by Nathwani et al. [41]. According to this method the utility function, the life quality index (LQI), is based on the income per capita (g) and on the life expectancy (e):

$$\mathrm{LQI} = g^{w} e^{(1-w)}$$

where w is the part of human life used for economic activities.

This method can be used to evaluate the investments in safety. An investment in safety, resulting in an increase of life expectancy (e) and a decrease of income (g), is acceptable if the LQI increases. An example of the use of the life quality method can be found in a study on investments in traffic safety [41]. The relation between utility, risk aversion and a monetary valuation can be shown in a monetary utility curve. van Gelder [42] gives an example.

8.4. Contingent valuation

This method is used in economics to value facilities, services or other benefits for which prices cannot be obtained from the market. A survey can reveal how much people are willing to pay, e.g. for safety measures. Such a study makes it possible to calculate the value of a statistical life (VoSL) by comparing the willingness to pay (WTP) and the expected number of fatalities ($E(N)$).

$$\mathrm{VoSL} = \frac{\mathrm{WTP} \cdot \mathrm{population}}{E(N)}$$

Pidgeon and Hopkins [43] give an example of such a study, concerning traffic safety. However, in this study no relation between the WTP and the expected number of fatalities could be found.

9. Case study: flood risk calculated with different risk measures

A case study has been performed to show the possible application of some of the described risk measures. In this study the flood risk is calculated for an existing polder in The Netherlands, with the following risk measures:

- *Individual risk*: compared with VROM and TAW standards.

- *Societal risk*: FN-curve, expected value of the number of fatalities, risk integral, total risk.
- *Economic risk*: FD-curve, expected value of the economic damage, economic optimisation.

A more extensive description of the case study and the models used is given in [44]. The studied area, "Betuwe, Tieler-en Culemborger Waarden" (BTCW), is situated in the eastern part of The Netherlands and measures about 80 by 25 km (Fig. 9). The polder is inhabited by approximately 360,000 persons and has an estimated economic value of about 40 billion Euros. The polder is threatened by river floods, in the north by the river Lek and in the south by the river Waal. Dikes that surround the whole polder (or the dike-ring, as it is called in The Netherlands) provide protection against high water.

The number of fatalities and the damage caused by a flood are mainly determined by the location of the initial breach. Therefore, the total dike-ring has been divided in fourteen distinct sections, each leading to a distinctly different flood pattern. For every section the probability of flooding has been determined with a model that takes the different failure modes of the dike into account (for example overtopping or instability of the dike). This results in an overall probability of flooding of the area of once in a thousand years. Breach simulations

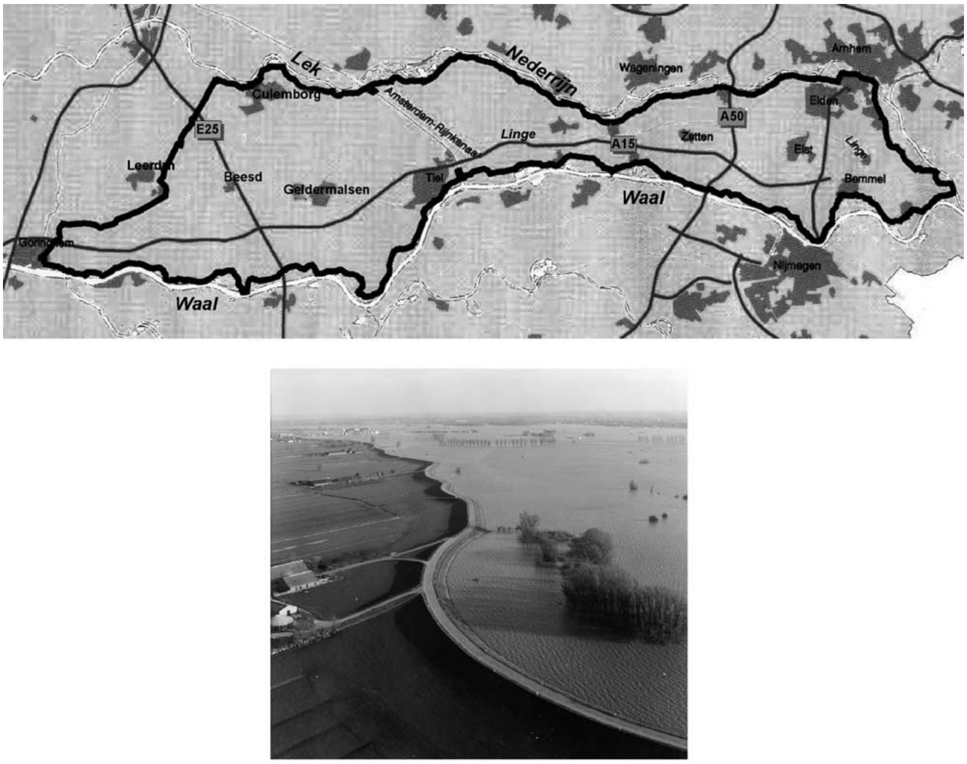

Fig. 9. The "Betuwe, Tieler-en Culemborger Waarden" area and a picture of the flood protection with dikes.

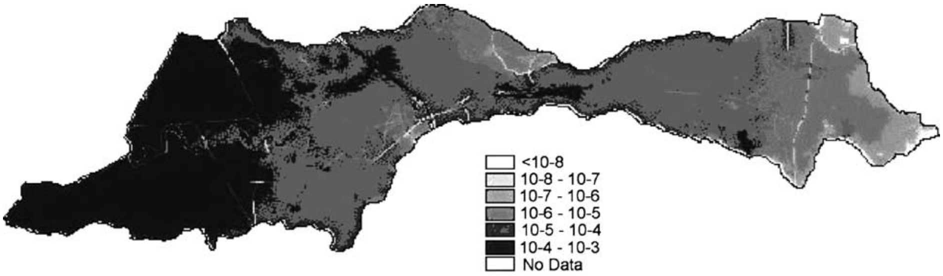

Fig. 10. Individual risk per year for the “Betuwe, Tieler-en Culemborger Waarden” polder.

of the flood pattern were made for every dike section. These results were used as input for a damage model (which calculates the number of fatalities and the economic damage for each flood). It has to be noted that the fatality modelling involves a lot of uncertainties, for instance, evacuation has not been included in the model. It is found that these uncertainties have a major impact on the magnitude of the calculated individual and societal risks.

9.1. Individual risk

The individual risk is calculated with the data acquired with the different flood simulations. For a certain flood, the probability of drowning at every location in the polder ($P_{d|i}$) is determined for all of the flood scenarios. The individual risk for every location (IR(x,y)) can be calculated by multiplication with the probability of the flood (P_i) and addition of the values for all n defined scenarios. The calculated individual risk levels for the area are shown in Fig. 10.

$$\mathrm{IR}(x, y) = \sum_{i=1}^{n} P_i P_{d|i}(x, y)$$

The IR of the polder can be compared with standards of VROM (accepting an individual risk of 10^{-6} per year) and TAW. The TAW proposed a β value of 0.1 for flood prone areas [3] which results in an acceptable IR of 10^{-5} per year. Fig. 11 shows in which areas the calculated IR for the polder would not be acceptable according to VROM and TAW standards (the dark areas exceed the limit).

9.2. Societal risk

The probability of occurrence and the number of fatalities are determined for every flood scenario. With this data a pdf of the number of fatalities is formed, from which a FN-curve can be derived. The FN-curve for the flooding of the BTCW is shown in Fig. 12.

The expected value of the number of fatalities per year and the standard deviation are derived from the pdf:

$$E(N) = 2.4\,(\text{fatalities/year}), \qquad \sigma(N) = 104.4\,(\text{fatalities/year})$$

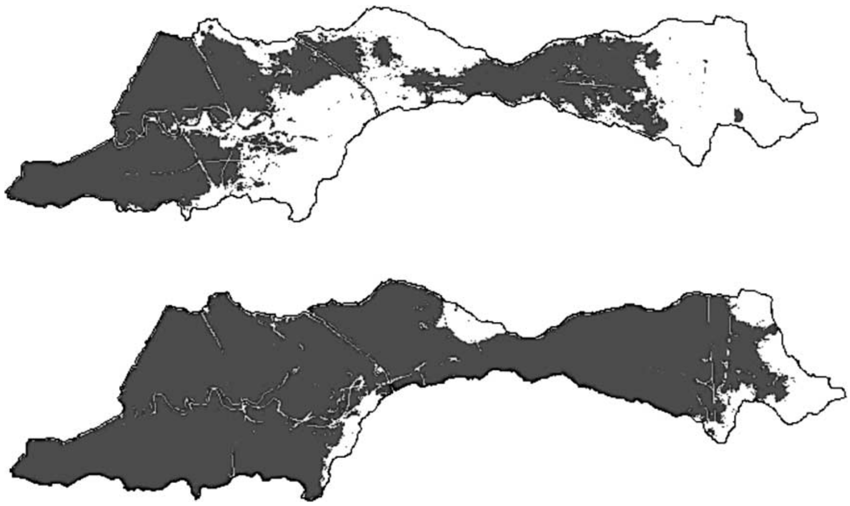

Fig. 11. Individual risk of the Betuwe polder compared with VROM standard and the TAW standard, dark areas exceed the limit.

The risk integral (RI) can be calculated with the expected value and the standard deviation of the number of fatalities [16]:

$$\mathrm{RI} = 0.5(E^2(N) + \sigma^2(N)) = 5452\ (\text{fatalities/year})^2$$

The total risk is also determined. For a value of the risk aversion index k of 3 this leads to:

$$\mathrm{TR} = E(N) + k\sigma(N) = 315.6\ (\text{fatalities/year})$$

The TAW criterion limits the total risk to a value of $\beta \cdot 100$, resulting in an acceptable total risk of 10 fatalities per year.

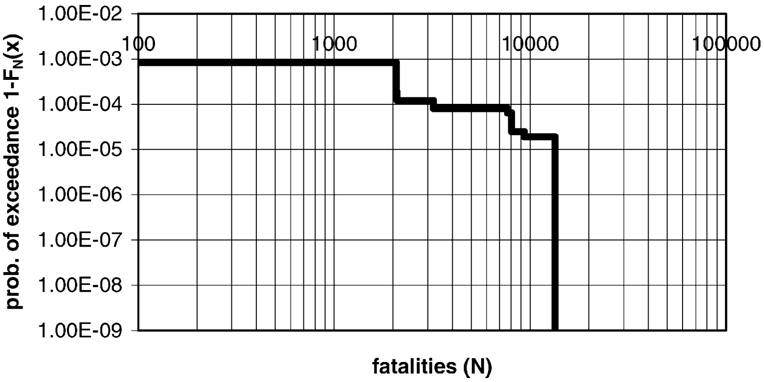

Fig. 12. FN-curve for the polder “Betuwe, Tieler-en Culemborger Waarden”.

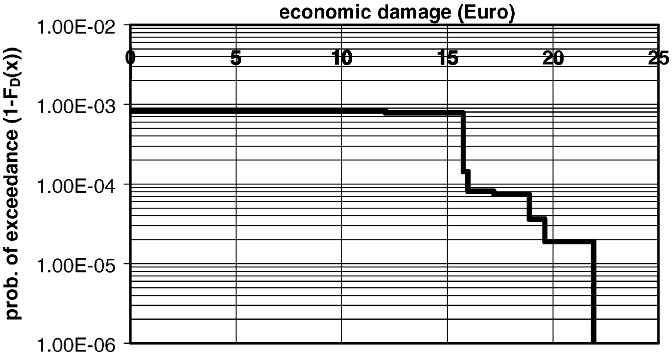

Fig. 13. FD-curve for the economic damage for the polder (x-axis on a non-logarithmic scale).

9.3. Economic risk

The economic risk is determined for every flood scenario. Information gathered from the different scenarios is used to form a pdf. From this pdf the FD-curve (Fig. 13) and the expected value of the economic damage can be derived.

$$E(D) = \int_0^\infty x\, f_D(x)\,\mathrm{d}x = 13.2\ (\text{million Euros})$$

The method of economic optimisation has also been applied for the BTCW polder. Fig. 14 shows the investments in dike improvement, the expected value of the economic damage and the sum of these two items, the total costs, as a function of the reliability index. This index can be converted to a probability of flooding. The economic optimum is found where the total costs are minimal. The situation occurs for a reliability index of 4, which equals a probability of flooding of $3.16 \cdot 10^{-5}$ per year (or once in 32,000 year).

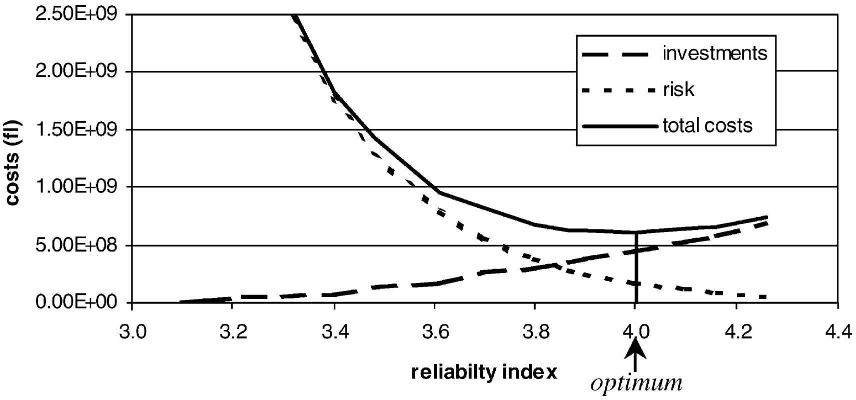

Fig. 14. Economic optimisation of the “Betuwe, Tieler-en Culemborger Waarden”.

10. Evaluation and summary of risk measures

This article gives an overview of various risk measures used in quantified risk analysis. A comprehensive overview of methods to quantify and limit risks arising from different sources is still missing in literature. This study concentrates on Dutch risk measurement experiences, mainly in the areas external safety and flood risk management. However, in order to give a more complete overview, some risk measures applied in other countries have also been included in the study. This summary indicates the different methods that can be used to quantify risks. A case study showed that it is possible to calculate the flood risk for an existing polder with various risk measures.

Table 2 summarises the most important characteristics of all the described risk measures. First, the name of the risk measure and the basic information needed for calculations are mentioned. After that, the table gives the used definitions of probabilities and consequences and the mathematical expression. The meaning of the symbols in the mathematical expression column can be found in the preceding text. The following columns show the field of application and, if any, the standard used. The last column refers to relevant literature. Some important aspects following from the overview and case study are outlined below.

10.1. Considered consequences

Most risk measures are limited by the fact that they consider only one type of consequences. This means that different characteristics are modelled into one number (or graph). Although, some argue that this oversimplifies the complex nature of risks, such a risk number can prove of significant use in the decision-making process. The overview shows that most risk measures are limited to considering fatalities, in the form of individual or societal risk. This is in line with the common view that the number of fatalities is the most important consequence of a disaster. The (potential) economic damage is also an important feature in decision-making concerning risks. Investments in safety can be compared with the decrease of economic risk that they cause. Both types of consequences, fatalities and economic damage, can be expressed in objective numbers, the number of fatalities in the economic damage in monetary units. This does not imply that risks should be judged only by the number of fatalities or the economic damage, but it does mean that these types of consequences are most suitable for a quantitative analysis.

It is possible to include the economic value of human life in an economic analysis of risks. Section 8 outlines different approaches for the economic valuation of human life. However, the economic valuation of human life raises numerous ethical and moral questions, because some people consider life invaluable.

Besides, some environmental risk measures and some methods, which consider the potential consequences, have been analysed. Also some integrated methods have been described that combine multiple types of consequences in one measure or a set of expressions. Notable is the fact that in the study of literature no risk measures for the number of injuries have been found, although this is often seen as an important type of consequence. Also for social disruption no uniform risk measures have been found. For these types of consequences a qualitative analysis may be more suitable.

Table 2
Overview of risk measures

Risk measure	Basis of calculation	Probability	Consequences	Mathematical expression	Field of application	Limit	Literature
Individual risk (IR)							
Individual risk (IR)	Probability of death for permanently present person	1 per year	Death of individual	$\mathrm{IR} = P_\mathrm{f}\, P_{\mathrm{d}\mid\mathrm{f}}$	Hazardous installations: The Netherlands (VROM)	$<10^{-6}$	[2,7,11]
IR—TAW	Probability of death for actually present person	1 per year	Death of individual	$\mathrm{IR} = P_\mathrm{f}\, P_{\mathrm{d}\mid\mathrm{f}}$	Studies, for example floods	$<\beta \cdot 10^{-4}$	[3,7]
IR—Bohnenblust	Probability of death for actually present person	1 per year	Death of individual	$\mathrm{IR} = P_\mathrm{f}\, P_{\mathrm{d}\mid\mathrm{f}}$	Studies railway safety in Germany	See Fig. 3	[4]
Individual risk—HSE	Probability of receiving a “dangerous dose” for a typical householder	1 per year	Receiving a “dangerous dose”	IR_HSE	UK (HSE): land use planning near hazardous installations	$<10^{-6}$ (boundary between tolerable and acceptable)	[6]
Societal risk							
Aggregated weighted risk (AWR)	Number of houses inside IR contour	1 per year	Fatalities	$\mathrm{AWR} = \iint_A \mathrm{IR}(x, y)\, h(x, y)\, \mathrm{d}x\, \mathrm{d}y$	The Netherlands: Schiphol	Stand still (no increase AWR)	[12]
Expected value of the number of fatalities from IR contours	Expected value from IR contours and population density	1 per year	Fatalities	$E(N) = \iint_A \mathrm{IR}(x, y)\, m(x, y)\, \mathrm{d}x\, \mathrm{d}y$	–	–	[13]
Scaled risk integral	IR, type of buildings, area, presence of persons	1/million year	Fatalities	$\mathrm{SRI} = \dfrac{P\, \mathrm{IR}_\mathrm{HSE}\, T}{A}$	HSE (UK): land use planning near hazardous installations	–	[14]
FN-curve	Probability density function of the number of fatalities	1 per year	Fatalities	$1 - F_N(x) = \int_x^\infty f_N(x)\, \mathrm{d}x$	International: hazardous activities (installations)	$1 - F_N(x) < (C/x^\alpha)$	[7,11,20, 21,24]
Expected value number of the number of fatalities *E*(*N*)	Probability density function of the number of fatalities	1 per year	Fatalities	$E(N) = \int_0^\infty x f_N(x)\, \mathrm{d}x$	US, Canada: dams	USBR: $<10^{-2}$ BC hydro: $<10^{-3}$	[22,23]
Risk integral	Probability density function of the number of fatalities	1 per year	Fatalities	$\mathrm{RI} = \int_0^\infty (1 - F_N(x)) x\, \mathrm{d}x$	HSE (UK): land use planning	–	[17]
Risk integral COMAH	Probability density function of the number of fatalities	1 per year	Fatalities	$\mathrm{RI}_\mathrm{COMAH} = \int_0^\infty x^\alpha f_N(x)\, \mathrm{d}x$	HSE (UK): land use planning near hazardous installations	–	[18]
Smets	Probability density function of the number of fatalities	1 per year	Fatalities	$\int_1^{1000} x^\alpha f_N(x)\, \mathrm{d}x$	–	$<10^{-2}$, for $\alpha = 2$	[11]
Bohnenblust	Probability density function of the number of fatalities	1 per year	Fatalities	$R_\mathrm{p} = \int_0^\infty x \varphi(x)\, f_N(x)\, \mathrm{d}x$	Studies railway safety in Germany	–	[4]
Kroon and Hoej	Probability density function of the number of fatalities	1 per year	Fatalities	$\int_0^\infty x^\alpha P(x)\, f_N(x)\, \mathrm{d}x$	OECD/PIARC study on tunnel safety	–	[19]
Total risk	Probability density function of the number of fatalities	1 per year	Fatalities	$\mathrm{TR} = E(N) + k\,\sigma(N)$	NL: studies External safety	$<\beta \cdot 100$	[7]

Table 2 (*Continued*)

Risk measure	Basis of calculation	Probability	Consequences	Mathematical expression	Field of application	Limit	Literature
Economic risk							
FD-curve	Probability density function of the economic damage	1 per year	Economic damage	$1 - F_D(x) = \int_x^\infty f_D(x)\,\mathrm{d}x$	Display various economic risks	Proposed by Jansen [28]	[27]
Expected value of the economic damage	Probability density function of the economic damage	1 per year	Economic damage	$E(D) = \int_0^\infty x f_D(x)\,\mathrm{d}x$	UK and NL: cost benefit analysis floods, US: dams	USBR: $E(D) <$ US$ 10,000	[29,30,31]
Economic optimisation	Minimise the sum of investments and economic risk	1 per year	Economic damage	$\min(C_{\mathrm{tot}}) = \min(I + E(D))$	NL: flood protection	Economic optimum	[25]
Economic optimisation and uncertainty	Minimise sum investments and risk including uncertainty	1 per year	Economic damage	$\min(\mu(C_{\mathrm{tot}}) + k\,\sigma(C_{\mathrm{tot}}))$	–	Economic optimum	[26]
Environmental risk							
Recovery time	Probability density function of recovery time (*T*) of the ecosystem	1 per year	Ecological damage (recovery time *T*)	$1 - F_T(x) = \int_x^\infty f_T(x)\,\mathrm{d}x$	NORSOK: oil platforms	$1 - F_T(x) < (0.05/T)$	[33]
Energetic impact index	Analysis of the amount of energy lost in the ecosystem	Indirect	Effect on ecosystem (J)	GPP lost = EPP + GPP′*T*	–	–	[34]
Integrated risk measures							
Bohnenblust	Analysis of risks for humans, economy and environment	1 per year	Fatalities, economic damage (environment)	$R_{\mathrm{m}} = \sum_{i=1}^{n} \varphi(C_i)\,\omega_i\,P_i\,C_i$	Studies railway safety in Germany	Economic optimum and IR limit	[4]
TAW	Individual risk proposed by TAW Total risk, Economic optimisation				Proposed by TAW for flood risk	IR $< \beta \cdot 10^{-4}$, TR $< \beta \cdot 100$, economic optimum	[3]
Merz et al.	Acceptable risk for various consequences determined with standard function	1 per year	Fatalities, injured, environment, economy	–	Kanton Solothurn (Swi)	See Fig. 8	[36]
Potential damage							
People at risk (PAR)	People at risk in the disaster area	–	People at risk	$\mathrm{PAR} = \iint_A m(x, y)\,\mathrm{d}x\,\mathrm{d}y$	Various fields	–	–
FPAR curve	People at risk in the disaster area	1 per year	People at risk	–	Risk assessment of dams	–	[35]
Potential economic damage	Economic value of the area	–	Economic damage	–	NL: flood protection	–	–
Methods including economic valuation of fatalities							
Economic optimisation including valuation of human life	Minimise sum investments and risk	1 per year	Economic damage and fatalities	$\min(C_{\mathrm{tot}}) = \min(I + E(D + Nd))$	–	Economic optimum	[3,38]
Life quality index	Decision-making about measures considering the effects for humans and economy	–	Influence on life expectancy and economy	$\mathrm{LQI} = g^w e^{1-w}$	Study on traffic safety	Increase in LQI	[41]

10.2. Risk: probability and consequences

In the analysis of risks both the magnitudes of the probabilities and of the consequences are of importance. A risk measure is defined as a mathematical function of the probability of an event and the consequences of that event. Most risk measures can thus be expressed with a mathematical formulation.

Although the formulae look different, Vrijling and van Gelder [16] showed that a majority of the risk measures can be expressed with similar characteristics. In general, the basis for the calculation of risks is the pdf. A more thorough inspection reveals that most societal risk measures can be derived from the pdf and that they are formulated as a measure for expected (dis)utility. Economic risk measures, such as the FD-curve and the expected value of the economic damage, are also based on the probability density function.

This was also shown in the case study, in which the flood risk of a polder in The Netherlands was calculated with different risk measures. After identifying the possible flood scenarios, the probabilities and consequences of the events were determined. This information was combined in a probability density function, which formed the basis for calculations with some societal and economic risk measures.

10.3. Risk aversion

In the societal perception of accidents an aversion to large accidents can be recognised, although the probability of such an accident is relatively low. Risk aversion can be taken into account in societal risk measures by weighing the expected value with a factor α (>1), by taking a risk aversion factor into account or by involving the standard deviation in the equation. The risk aversion to larger accidents can also be also be modelled in the standard, by accepting larger consequences with a relatively smaller probability.

10.4. Field of application and limits

This article also gives some background about the fields of application of the measures and the standards used in these fields. From literature it can be concluded that the use of some risk measures is widespread over the world. The FN-curve, for instance, is used in various countries to express and limit the risks of hazardous activities. For other measures no actual use is reported, neither have standards been proposed. The determination of the limit and the upholding of the standards are often governmental tasks. In The Netherlands, risks are limited by the Ministry of Housing, Spatial Planning and Environment, in the UK by the Health and Safety Executive. An extensive study has been undertaken to compare current risk assessment practices and regulations across different industries and different EU countries [45]. Limits can also be determined by (utility) companies, for example, for dams in Canada, by British Columbia Hydro. Besides quantitative limits, other principles can be used for the regulation of risks. In The Netherlands, risks of the national airport Schiphol have been limited with the stand still principle, which deems no further increase of risks acceptable. The use of other than risk based approaches, for instance generic safety distances and consequence based approach, for land use planning in several countries in the European Union has been described by Christou et al. [46]. The problem of the acceptable level of risk can also be formulated as an economic decision problem. Economic risks are often weighted in the framework of a cost benefit analysis. Another method is economic optimisation, which considers the investments in safety measures and the reduction of economic risk.

This study has mainly focused on the outcomes of the risk assessment procedures. Benchmark exercises studies have also indicated the importance of the risk analysis process and the models used. A benchmark study by Lauridsen et al. [47], considering a risk analysis for an ammonia storage facility, has indicated significant differences between the participants, both in frequency and consequence assessments, which can lead to variations in the final risk results. However, a recent benchmark study in The Netherlands [48] indicates lower variations and shows that a certain level of harmonisation with guidelines improves the coherence of the results.

10.5. Recommendations

The answer to the question “how safe is safe enough?” should come from a broad judgement of all relevant aspects. Therefore, co-operation with other fields of science in the study of risks and risk measures is necessary. For instance, psychological studies can be of interest in determining societal risk acceptance and the accompanying quantitative criteria. Finally, it has to be stated that this overview is far from a complete review of quantitative risk measures.

Disclaimer

Any opinions expressed in this paper are those of the authors and do not necessarily reflect the position of the Dutch Ministry of Transport, Public Works and Water Management.

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